Find the mistake in an inequality problem

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Question

Given $1 < x < 6$, $5 < y < 7$, find the range of $\dfrac{y}{x+y}$.

Find and give reasons for which of the following calculations are incorrect or unsatisfactory:

1.

$1 < x < 6$ …. (1)

$5 < y < 7$ …. (2)

(1) + (2), $6 < x + y < 13$ …. (3)

1/(3), $\dfrac{1}{13} < \dfrac{1}{x+y} < \dfrac{1}{6}$ …. (4)

(2)×(4), $\dfrac{5}{13} < \dfrac{y}{x+y} < \dfrac{7}{6}$

2.

$1 < x < 6$ …. (1)

$5 < y < 7$ …. (2)

1/(2), $\dfrac{1}{7} < \dfrac{1}{y} < \dfrac{1}{5}$ …. (3)

(1) ×(3), $\dfrac{1}{7} < \dfrac{x}{y} < \dfrac{6}{5}$ …. (4)

(4) + 1, $\dfrac{1}{7} + 1 < \dfrac{x}{y} + 1 < \dfrac{6}{5} + 1$

$\dfrac{8}{7} < \dfrac{x+y}{y} < \dfrac{11}{5}$ …. (5)

1/(5), $\dfrac{5}{11} < \dfrac{y}{x+y} < \dfrac{7}{8}$

3.

$1 < x < 6$ …. (1)

$5 < y < 7$ …. (2)

(1) + (2), $6 < x + y < 13$ …. (3)

1/(3), $\dfrac{1}{13} < \dfrac{1}{x+y} < \dfrac{1}{6}$ …. (4)

(1) ×(4), $\dfrac{1}{13} < \dfrac{x}{x+y} < 1$ …. (5)

(5) – 1, $1 - 1 < 1 - \dfrac{x}{x+y} < 1 - \dfrac{1}{13} \quad \Rightarrow 0 < \dfrac{(x+y)-x}{x+y} < \dfrac{12}{13}$

$0 < \dfrac{y}{x+y} < \dfrac{12}{13}$

4. $1 < x < 6$ …. (1)

$5 < y < 7$ …. (2)

The table shows the value of $\dfrac{y}{x+y}$:

y / x	1	2	3	4	5	6
5	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$	$\frac{5}{9}$	$\frac{5}{10}$	$\frac{5}{11}$
6	$\frac{6}{7}$	$\frac{6}{8}$	$\frac{6}{9}$	$\frac{6}{10}$	$\frac{6}{11}$	$\frac{6}{12}$
7	$\frac{7}{8}$	$\frac{7}{9}$	$\frac{7}{10}$	$\frac{7}{11}$	$\frac{7}{12}$	$\frac{7}{13}$

$\therefore \quad \dfrac{5}{11} < \dfrac{y}{x+y} < \dfrac{7}{8}$

5. $1 < x < 6$ …. (1)

$5 < y < 7$ …. (2)

(1) ×(3), $5 < xy < 42$ …. (3)

$(2)^2$, $25 < y^2 < 49$ …. (4)

(3) + (4), $30 < y(x+y) < 91$ …. (5)

1/(5), $\dfrac{1}{91} < \dfrac{1}{y(x+y)} < \dfrac{1}{30}$ …. (6)

(3) ×(6), $\dfrac{5}{91} < \dfrac{xy}{y(x+y)} < \dfrac{42}{30}$

$\therefore \quad \dfrac{5}{91} < \dfrac{y}{x+y} < \dfrac{42}{30}$

6. $1 < x < 6$ …. (1)

$5 < y < 7$ …. (2)

1/(2), $\dfrac{1}{7} < \dfrac{1}{y} < \dfrac{1}{5}$ …. (3)

(1) ×(3), $\dfrac{1}{7} < \dfrac{x}{y} < \dfrac{6}{5}$ …. (4)

$\dfrac{y}{x+y} = \dfrac{\frac{y}{y}}{\frac{x+y}{y}} = \dfrac{1}{\frac{x}{y}+1} > \dfrac{1}{\frac{6}{5}+1} = \dfrac{5}{11}$ and $\dfrac{y}{x+y} = \dfrac{1}{\frac{x}{y}+1} < \dfrac{1}{\frac{1}{7}+1} = \dfrac{7}{8}$

$\therefore \quad \dfrac{5}{11} < \dfrac{y}{x+y} < \dfrac{7}{8}$